No black hole information puzzle in a relational universe

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The introduction of a relational time in quantum gravity naturally implies that pure quantum states evolve into mixed quantum states. We show, using a recently proposed concrete implementation, that the rate at which pure states naturally evolve into mixed ones is faster than that due to collapsing into a black hole that later evaporates. This is rather remarkable since the fundamental mechanism for decoherence is usually very weak. Therefore the "black hole information puzzle" is rendered de-facto unobservable.

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Every physicist notes, upon first being introduced to quantum mechanics, that the role of the variable "t" is somewhat artificial. One is expected to believe the existence of a perfectly classical external clock to the system in observation and to treat time as a classical variable. This is clearly an approximation, that cannot ultimately be entirely accurate. After all, every clock will have some quantum fluctuations. This is particularly true in situations where quantum gravity effects may be of interest, since it is hard to imagine suitably "external and classical" clocks will be available at the relevant energies.

How does one do quantum mechanics without classical clocks? The idea consists in promoting all variables in the problem, including those one may wish to choose as the clock variables, to quantum operators. One then computes conditional probabilities for the variables one wishes to observe assuming the variables chosen as clock variables take certain desired values. If there is a single variable that (at least for a while) behaves approximately classically, we can call that variable t. Then the conditional probability of other variables of interest taking a given value x when t takes a certain value t_T, $P(x|t_T)$, will approximately satisfy a Schrödinger equation. When there is no variable that is a good classical clock the relational approach is still valid and the conditional probabilities exist, but the interpretation of the probabilities as constituting a traditional Schrödinger picture does not.

This framework appears remarkably natural to address the "problem of time" in quantum gravity and was discussed in this context by Page and Wootters [1]. Unfortunately, in the case of general relativity, the presence of the constraints causes trouble. As discussed in great detail by Kuchař [2] it is not possible to have a meaningful relational description in traditional canonical quantum gravity. Since all observables of the theory are also "perennials" (they have vanishing Poisson brackets with the Hamiltonian constraint) none of them is suitable as a "clock". Page and Wootters attempt to bypass this by choosing to build the relational framework in terms of quantities that do not have vanishing Poisson brackets with the constraints (that is, they choose to work at the kinematical level.) However, quantum mechanically, the states that are annihilated by the constraints are expected to be distributional within the space of kinematical states and do not lead to a good probabilistic interpretation. Kuchař showed in model systems that the resulting propagators are proportional to the Delta function and therefore "they don't propagate" [2].

We have recently introduced new discretization scheme for general relativity [3]. The resulting discrete theory contains solutions that are arbitrarily close to those of general relativity, yet is free of constraints. Therefore the major conceptual objections to using the relational approach to solving the problem of time are removed. We have discussed in some detail the application of this approach in ref. [4].

In quantum mechanics with a relational time, since the clock is not perfect, it is inevitable that pure quantum states evolve into mixed quantum states. This is what brings us to the black hole information paradox. In 1973 Hawking [5] noted using quantum field theory on curved space-time techniques, that a black hole with mass M will emit radiation as if it were at a temperature,

$$T = \frac{\hbar}{8\pi k M} \tag{1}$$

where k is Boltzmann's constant. Black holes are therefore not ever-living anymore since they lose mass through the emitted radiation. It is expected that eventually black holes evaporate completely and the only thing left is the outgoing radiation. This brings about the black hole information puzzle. Suppose one prepares a pure quantum state and collapses it to form a black hole. Eventually all that is left is a thermal state of the outgoing radiation, which is a mixed state. Therefore the pure state has evolved into a mixed state.

In a world where unitarity rules, this is a problem. Unitarity rules in the idealized world of Schrödinger quantum mechanics, with its perfect clocks. In the more realistic world of quantum mechanics with a relational time, the lack of perfect clocks means pure states evolve into mixed states naturally. The black hole information "problem" will only arise if somehow black holes were more efficient at destroying the coherence of pure states than the lack

of a perfect clock. Here we will attempt to estimate if this is the case.

Several alternatives have been proposed as solutions of the black hole information problem. A good brief summary can be found in the paper by Giddings and Thorlacius [6]. Hawking [7] had proposed that unitarity is lost in quantum mechanics due to interactions with virtual black holes forming the “space-time foam”. This approach has been criticized on the grounds that it leads to the loss of the conservation of energy [8]. It should be noted that our proposal, although it has in common with Hawking's that it leads to pure states evolving into mixed states, does conserve energy due to the particular form of decoherence (it is a Lindblad [9] type of evolution, but it is governed by the Hamiltonian and automatically guarantees its conservation, see [4].) It should be noted that other effects, like the production of virtual pairs of black holes [8] or the entanglement of the clock and the system upon evolution could lead to lack of conservation of energy, but to a first approximation energy is conserved in our approach. A second alternative that was proposed as a solution to the paradox is that the black hole does not evaporate completely, and a “remnant” containing all the information is left. A challenge for proponents of this approach is to find a satisfactory description of the remnants and to avoid infinite production rates [10]. Finally, a third avenue is to attempt to find a way to send the information out with the outgoing radiation in a process similar to quantum teleportation. A main concern is to find dynamics that is non-local enough to achieve the teleportation. Susskind has argued that string theory is non-local enough in this sense [11]. A very recent and attractive proposal along these lines is due to Horowitz and Maldacena [12] in which they propose giving a boundary condition at the singularity that transfers information to the outgoing radiation in a process similar to quantum teleportation. Recently Gottesman and Preskill have shown that one may still face non unitarities within this scheme [13].

We now proceed to describe our proposal. We have recently presented a detailed calculation of the rate of decoherence of a pure state in the relational picture emerging from discrete quantum gravity. The result is that the time for losing complete coherence is given by [4],

$$t_{\text{coherence loss}} \sim \frac{1}{\omega^2 t_{\text{Planck}}}, \tag{2}$$

where ω is the frequency associated with the spread of energy of the possible states of the system under study. For a system with two energy levels, it would be the transition frequency. The details of the derivation of this formula are lengthy [4, 14] and not germane to what we are discussing, but it can be remarked that the formula appears quite natural: a system with states with big energy differences will tend to decohere more rapidly, and the natural scale for decoherence due to quantum gravity ought to be the Planck scale. So one could arrive to the preceding formula just on dimensional analysis considerations. The resulting effect is very small. It is conceivable that it could be observed experimentally in the future, but it appears inaccessible to current technology [4].

If we now consider a black hole, the energy fluctuations in the system are characterized by, $kT = \hbar/8\pi M$, which yields a frequency, $\omega = kT/\hbar = 1/8\pi M$ (we use units in which $G = c = 1$.) It is clear that extrapolating our formula, which was derived for a quantum mechanical system, to a situation like a black hole can only be considered as a first approach, and to give a rough order-of-magnitude estimation of the effect. More detailed modeling is clearly needed, but at the moment a reliable model with a detailed microscopic description of the black hole seems beyond the reach of the state of the art of the field.

Combining with our formula for the decoherence time we find that the system “black hole plus pure state” just due to the possible quantum fluctuations of the black hole radiation, will decohere in a time (from the point of view of an external observer at a fixed distance from the hole),

$$t_{\text{BH deco}} \sim \frac{(8\pi M)^2}{t_{\text{Planck}}}. \tag{3}$$

On the other hand, the pure state will lose coherence through the complete evaporation of the black hole in a time [15],

$$t_{\text{evaporation}} = (M/M_p)^3 t_{\text{Planck}} \tag{4}$$

For the black hole evaporation to be faster than the natural decoherence of the state, we have,

$$M < (8\pi)^2 M_{\text{Planck}}, \tag{5}$$

that is, the black hole should be smaller than 631 Planck masses. For such microscopic black holes the semiclassical picture in which Hawking radiation is emitted is not valid and therefore one cannot formulate the information paradox in the traditional sense. An analysis of this case would require a full quantum gravity calculation.

The reader may wonder why is the mass limit so small? The reason is that the loss of unitarity of the relational theory is produced by the fluctuations in energy of the radiation emerging from the black hole, and those fluctuations increase when the black hole is smaller. That is, a smaller black hole will evaporate faster, but it will also decohere faster in the relational picture. It should also be noted that our approach does not preclude the loss of information through tunneling to another hypothetical semiclassical region beyond the singularity inside the black hole. In fact it provides a natural mechanism to achieve such regions [16].

Summarizing, we have argued that the correct way to view quantum mechanics in a realistic universe where

perfect classical clocks do not exist is the relational approach. This approach has recently been incorporated into the quantum gravity context via the consistent discretization methods. With the introduction of a relational time pure states evolve into mixed states naturally, and we have here shown that they do it fast enough that the black hole information puzzle does not appear. It should be noted that in spite of this, the rate of loss of coherence is really minute for everyday phenomena. For instance the decoherence time for a Solar sized black hole is 10^{41} years. However, the evaporation times are even larger, about 10^{66} years.

To conclude, quantum mechanics with a relational time leads to a lack of coherence that is minute and therefore does not interfere with most of the physics we know, but it is large enough to eliminate the black hole information puzzle.

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[1] D. N. Page and W. K. Wootters, Phys. Rev. D **27**, 2885 (1983)

[2] K. Kuchař, "Time and interpretations of quantum gravity", in "Proceedings of the 4th Canadian conference on general relativity and relativistic astrophysics", G. Kunstatter, D. Vincent, J. Williams (editors), World Scientific, Singapore (1992). [Available at http://www.phys.lsu.edu/faculty/pullin/kvk.pdf]

[3] C. Di Bartolo, R. Gambini, J. Pullin, Class. Quan. Grav. 19, 5475 (2002); R. Gambini and J. Pullin, Phys. Rev. Lett. 90, 021301, (2003); R. Gambini, R.A. Porto and J. Pullin, in "Recent developments in gravity", K. Kokkotas, N. Stergioulas, World Scientific, Singapore, (2003) [arXiv:gr-qc/0302064].

[4] R. Gambini, R. A. Porto and J. Pullin, Class. Quan. Grav. **21** L51 (2004).

[5] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[6] S. Giddings, L. Thorlacius, in "Particle and nuclear astrophysics and cosmology in the next millennium", E. Kolb (editor), World Scientific, Singapore (1996) [arXiv:astro-ph/9412046].

[7] S. W. Hawking, Commun. Math. Phys. **87**, 395 (1982).

[8] J. Ellis, J. Hagelin, D.V. Nanopoulos, and M. Srednicki, Nucl. Phys. B241 (1984) 381; T. Banks, M.E. Peskin, and L. Susskind, Nucl. Phys. B244 (1984) 125.

[9] G. Lindblad, Commun. Math. Phys. 48, 119 (1976)

[10] G. Gibbons, in "Fields and geometry", ed. A. Jadczyk (World Scientific, 1986); D. Garfinkle and A. Strominger, Phys. Lett. B256 (1991) 146; A. Strominger and S. Trivedi, Phys. Rev. **D48**, 5778 (1993).

[11] L. Susskind, Phys. Rev. Lett. 71 (1993) 2367; Phys. Rev. D49 (1994) 6606.

[12] G. T. Horowitz and J. Maldacena, JHEP **0402** 008 (2004).

[13] D. Gottesman and J. Preskill, JHEP **0403** 026 (2004).

[14] R. Gambini, R. Porto, J. Pullin, New. J. Phys. **6**, 45 (2004).

[15] C. Kiefer, in "Decoherence and entropy in complex systems", H.-T. Elze (editor), Springer-Verlag, New York (2003) [arXiv:gr-qc/0304102].

[16] R. Gambini, J. Pullin, Int. J. Mod. Phys. **D12** 1775 (2003).